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2. REPORT DATE	3. REPORT TYPE AND DATES COVERED Final/1 Jun 89 - 30 Nov 90
4. TITLE AND SUBTITLE Theoretical Investigations of Chaotic Dynamics (U)	5. FUNDING NUMBERS 61102F 2304/A4
6. AUTHOR(S) Professor James Yorke	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) University of Maryland Institute for Physical Science & Technology College Park, MD 20742-2431	8. PERFORMING ORGANIZATION REPORT NUMBER AFOSR-TR- 91 0679
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) AFOSR/NM Bldg 410 Bolling AFB DC 20332-6448	10. SPONSORING/MONITORING AGENCY REPORT NUMBER AFOSR-89-0401
11. SUPPLEMENTARY NOTES	
12a. DISTRIBUTION/AVAILABILITY STATEMENT Approved for Public Release; Distribution Unlimited	12b. DISTRIBUTION CODE UL

13. ABSTRACT (Maximum 200 words)

One of the outstanding problems of chaotic dynamics has been to show that chaos develops monotonically as the parameter is varied, for some systems. Results along this line have been very few. An overview of these results has been given in the proposal for this funding period. Kan and Yorke have discovered results in two dimensions which clearly indicate the situation is far worse than previously believed. Their results require some mild nondegeneracy conditions which shall not be spelled out in detail here. Their results are for diffeomorphisms that depend on a parameter. They show that monotonicity never occurs in two dimensions as the parameter varies, except in the most trivial situations. In (KY) these results have been written for a special prototype example which seems quite typical. This example has nice simple choices of coordinates, and analysis is facilitated. Establishment of the full result was much more difficult and the analysis has been carried out in (KKY)

91-13721

14. SUBJECT TERMS			15. NUMBER OF PAGES 0
			16. PRICE CODE
17. SECURITY CLASSIFICATION OF REPORT UNCLASSIFIED	18. SECURITY CLASSIFICATION OF THIS PAGE UNCLASSIFIED	19. SECURITY CLASSIFICATION OF ABSTRACT UNCLASSIFIED	20. LIMITATION OF ABSTRACT SAR

UNIVERSITY OF MARYLAND

FINAL REPORT

FOR THE

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

OF THE RESEARCH GRANT AFOSR-89-0401

ENTITLED

THEORETICAL INVESTIGATIONS OF CHAOTIC DYNAMICS

PRINCIPAL INVESTIGATOR

JAMES A. YORKE

FUNDING PERIOD

1 JUN 89 - 30 NOV 90

AUGUST 1991

PREFACE.

This final report for the Air Force Office of Scientific Research (research grant AFOSR 01-5-28225) entitled "Theoretical Investigations of Chaotic Dynamics" of the period December 1, 1988 - November 30, 1990 consists of two parts. In part I, an overview of the accomplished scientific activities is presented, and part II is devoted to abstracts of all papers that were (almost) finished in the actual funding period. Some papers in part I refer to the incorrect (outdated) AFOSR grant 81-0217, which was a predecessor of the current grant.

Note: Z.P. You received his Ph.D. degree in Mathematics in August 1991 from the University of Maryland with J.A. Yorke as dissertation advisor.

PART I.

ACTIVITIES

1. HOW CHAOS DEVELOPS AS A PARAMETER IS VARIED

One of the outstanding problems of chaotic dynamics has been to show that chaos develops monotonically as the parameter is varied, for some systems. Results along this line have been very few. An overview of these results has been given in the proposal for this funding period. Kan and Yorke have discovered results in two dimensions which clearly indicate the situation is far worse than previously believed. Their results require some mild nondegeneracy conditions which shall not be spelled out in detail here. Their results are for diffeomorphisms that depend on a parameter. They show that monotonicity <u>never</u> occurs in two dimensions as the parameter varies, except in the most trivial situations. In [KY] these results have been written for a special prototype example which seems quite typical. This example has nice simple choices of coordinates, and analysis is facilitated. Establishment of the full result was much more difficult and the analysis has been carried out in [KKY].

[KY] I. Kan and J.A. Yorke. Antimonotonicity: concurrent creation and annihilation of periodic orbits. Bulletin (New Series) Amer. Math. Soc. 23 (1990), 469-476.

[KKY] I. Kan, H. Koçak, and J.A. Yorke. Antimonotonicity: concurrent creation and annihilation of periodic orbits. Preprint University of Maryland, August 1990. To appear in Annals of Math.

2. NUMERICAL METHODS FOR CHAOTIC DYNAMICAL SYSTEMS

2A. STRADDLE ORBITS. Examples are common in dynamical systems in which there are regions containing chaotic sets that are not attractors. If almost every trajectory eventually leaves some region, but the region contains a chaotic set, then typical trajectories will behave chaotically for a while and then will leave the region, and so we will observe chaotic transients. Such regions are called transient regions. Systems with horseshoes have such regions as do systems with fractal basin boundaries, as does the Hénon map for suitably chosen parameters. In [NY1] we presented a numerical method for finding trajectories which will stay in such transient regions for arbitrarily long periods of time, and it leads to a "saddle straddle trajectory". Furthermore, a refined procedure for finding accessible trajectories on the chaotic saddle has also been discussed. In [NY2] these numerical methods are shown to be valid for hyperbolic systems. The examples in [NY1] illustrate that the procedure works well on computers for long periods of time, even when the system lacks hyperbolicity.

In dynamical systems examples are common in which two or more attractors coexist, and in such cases the basin boundary is nonempty. When the basin boundary is fractal (that is, it has a Cantor-like structure) a relatively small subset of a fractal basin boundary is said to be "accessible" from a basin. However, these accessible points play an important role in the dynamics, and especially, in showing how the dynamics change as parameters are varied. In [NY3] a numerical procedure is presented that enables to produce trajectories lying in this accessible set on the basin boundary, and it is proven that this procedure is valid in certain hyperbolic systems.

[NY1] H.E. Nusse and J.A. Yorke. A procedure for finding numerical trajectories on chaotic saddles. Physica D 36 (1989), 137-156.

[NY2] H.E. Nusse and J.A. Yorke. Analysis of a procedure for finding numerical trajectories close to invariant chaotic saddle hyperbolic sets. Ergodic Theory and Dynamical Systems 11 (1991), 189-208.

[NY3] H.E. Nusse and J.A. Yorke. A numerical procedure for finding accessible trajectories on basin boundaries. Preprint Univ. of Maryland, December 1989. To appear in Nonlinearity (1991).

2B. MANIFOLDS. For years it has been known that in two-dimensional dynamical systems the stable and unstable manifold of a saddle fixed point can locally be approximated by a line segment. Furthermore, the unstable (stable) manifold of the fixed point is the union of the forward (backward) images of the local stable (unstable) manifold which is a curve segment containing the fixed point. In [YKY] a numerical procedure is described for computing one-dimensional stable and unstable manifolds of fixed points (periodic points) for diffeomorphisms of the n-dimensional Euclidian space. In particular, it has been showed that a plot of the computed curve coincides with the true curve within the resolution of the display. A second procedure is described to minimize the amount of computation of parts of the curve that lie outside a region of interest. The method is applied to compute the one-dimensional stable and unstable manifolds of some different systems.

[YKY] Z. You, E.J. Kostelich, and J.A. Yorke: Calculating stable and unstable manifolds. Preprint University of Maryland, January 1991. To appear in International Journal of Bifurcation and Chaos.

3. TOPOLOGICAL AND ANALYTICAL METHODS FOR DYNAMICAL SYSTEMS

3A. WADA. In dynamical systems examples are common in which two or more attractors coexist, and in such cases the basin boundary is nonempty. In [KeY] situations have been described in which there are several basins of attraction (more than two) with the Wada property, namely that each point that is on the boundary of one basin of attraction is on the boundary of all basins of attraction. It has been argued that such situations arise even in studies of the forced damped pendulum.

[KeY] J. Kennedy and J.A. Yorke: Basins of Wada. Preprint University of Maryland, December 1990. To appear in Physica D.

3B. DIMENSION. McDonald, Grebogi, Ott, and Yorke introduced the uncertainty dimension as a quantative measure for final state sensitivity in a system. It was conjectured that the box-counting dimension equals the uncertainty dimension for basin boundaries in typical dynamical systems. This conjecture has been established in [NY]; the main result is that the box-counting dimension, the uncertainty dimension and the Hausdorff dimension are all equal for the basin boundaries of one and two dimensional systems, which are uniformly hyperbolic on their basin boundary.

[NY] H.E. Nusse and J.A. Yorke: The equality of fractal dimension and uncertainty dimension for certain dynamical systems. Preprint University of Maryland, December 1990. Submitted for publication.

3C. EMBEDOLOGY. Mathematical formulations of the embedding methods commonly used for the reconstruction of attractors from data series are discussed. Embedding theorems, based on previous work by H. Whitney and F. takens, are established for compact subsets A of the k-dimensional Euclidian space $\mathbb{R}^k$. If n is an integer larger than twice the box-counting dimension of A, then almost every map from $\mathbb{R}^k$ to $\mathbb{R}^n$, in the sense of prevalence, is one-to-one on A. If A is a chaotic attractor of a typical dynamical system, then the same is true for almost every delay-coordinate map from $\mathbb{R}^k$ to $\mathbb{R}^n$.

[SYC] T. Sauer, J.A. Yorke and M. Casdagli: Embedology. Preprint University of Maryland, January 1991. To appear in Journal of Statistical Physics.

3D. ACCESSIBLE SADDLES. The paper [AY] (version 1988) was described in the final scientific report of October 1988. However, this paper was completely revised in 1989.

[AY] K.T. Alligood and J.A. Yorke: ACCESSIBLE SADDLES ON FRACTAL BASIN BOUNDARIES. Preprint University of Maryland, November 1989. To appear in Ergodic Theory and Dynamical Systems

PART II.

PUBLICATIONS AND ABSTRACTS

H. E. Nusse and J. A. Yorke: A procedure for finding numerical trajectories on chaotic saddles. Physica 36D (1989), 137-156

H. E. Nusse and J. A. Yorke: ANALYSIS OF A PROCEDURE FOR FINDING NUMERICAL TRAJECTORIES CLOSE TO CHAOTIC SADDLE HYPERBOLIC SETS. Ergodic Theory and Dynamical Systems 11 (1991), 189-208.

ABSTRACT. In dynamical systems examples are common in which there are regions containing chaotic sets that are not attractors, e.g. systems with horseshoes have such regions. In such dynamical systems one will observe chaotic transients. An important problem is the "Dynamical Restraint Problem": Given a region that contains a chaotic set but contains no attractor, find a chaotic trajectory numerically that remains in the region for an arbitrarily long period of time. We present two procedures ("PIM triple procedures") for finding trajectories which stay extremely close to such chaotic sets for arbitrarily long periods of time.

K.T. Alligood and J.A. Yorke: ACCESSIBLE SADDLES ON FRACTAL BASIN BOUNDARIES. Preprint University of Maryland, November 1989. To appear in Ergodic Theory and Dynamical Systems

ABSTRACT. For a homeomorphism of the plane, the basin of attraction of a fixed point attractor is open, connected, and simply-connected, and hence is homeomorphic to an open disk. The basin boundary, however, need not to be homeomorphic to a circle. When it is not, it can contain periodic orbits of infinitely many different periods.

Certain points on the basin boundary are distinguished by being accessible (by a path) from the interior of the basin. For an orientation-preserving homeomorphism, the accessible boundary points have a well-defined rotation number. We prove that this rotation number is rational if and only if there are accessible periodic orbits. In particular, if the rotation number is the reduced fraction p/q, then every accessible periodic orbit has minimum period q. In addition, if the periodic points are hyperbolic, then every accessible point is on the stable manifold of an accessible periodic point.

H. E. Nusse and J. A. Yorke: A NUMERICAL PROCEDURE FOR FINDING ACCESSIBLE TRAJECTORIES ON BASIN BOUNDARIES. Preprint University of Maryland, December 1989. To appear in Nonlinearity (1991)

ABSTRACT. In dynamical systems examples are common in which two or more attractors coexist, and in such cases the basin boundary is nonempty. The basin boundary is either smooth or fractal (that is, it has a Cantor-like structure). When there are horseshoes in the basin boundary, the basin boundary is fractal. A relatively small subset of a fractal basin boundary is said to be "accessible" from a basin. However, these accessible points play an important role in the dynamics, and especially, in showing how the dynamics change as parameters are varied. The purpose of this paper is to present a numerical procedure that enables us to produce trajectories lying in this accessible set on the basin boundary, and we prove that this procedure is valid in certain hyperbolic systems.

I. Kan and J.A. Yorke. ANTIMONOTONICITY: CONCURRENT CREATION AND ANNIHILATION OF PERIODIC ORBITS. Bulletin (New Series) Amer. Math. Soc. 23 (1990), 469-476.

I. Kan, H. Koçak, and J.A. Yorke. ANTIMONOTONICITY: CONCURRENT CREATION AND ANNIHILATION OF PERIODIC ORBITS. Preprint University of Maryland, August 1990. To appear in Annals of Math.

ABSTRACT. One-parameter families f_λ of diffeomorphisms of the Euclidian plane are known to have a complicated bifurcation pattern as λ varies near certain values, namely where homoclinic tangencies are created. In the first paper we argue and in the second paper we establish, that the bifurcation pattern is much more irregular than previously reported. Our results contrast with the monotonicity result for the well-understood one-dimensional family $g_\lambda(x) = \lambda x(1-x)$, where it is known that periodic orbits are created and never annihilated as λ increases. We show that this monotonicity in the creation of periodic orbits never occurs for one-parameter family of area contracting three times continuously differentiable diffeomorphisms of the Euclidian plane, excluding certain technical degenerate cases where our analysis breaks down. It has been shown that in each neighborhood of a parameter value

at which a homoclinic tangency occurs, there are either infinitely many parameter values at which periodic orbits are created or infinitely many at which periodic orbits are annihilated. We show that there are both infinitely many values at which periodic orbits are created, and infinitely many at which periodic orbits are annihilated. We call this phenomenon antimonotonicity.

J. Kennedy and J.A. Yorke: BASINS OF WADA. Preprint University of Maryland, December 1990. To appear in Physica D.

ABSTRACT. We describe situations in which there are several regions (more than two) with the Wada property, namely that each point that is on the boundary of one region is on the boundary of all. We argue that such situations arise even in studies of the forced damped pendulum, where it is possible to have three attractors coexisting, and the three basins of attraction have the Wada property.

H.E. Nusse and J.A. Yorke: THE EQUALITY OF FRACTAL DIMENSION AND UNCERTAINTY DIMENSION FOR CERTAIN DYNAMICAL SYSTEMS. Preprint University of Maryland, December 1990. Submitted for publication.

ABSTRACT. McDonald, Grebogi, Ott, and Yorke introduced in the paper "Fractal basin boundaries" (Physica D 17 (1985), 125-153) the uncertainty dimension as a quantative measure for final state sensitivity in a system. In that paper and in the paper "A dynamical meaning of fractal dimension" (Transactions Amer. Math. Soc. 292 (1985), 695-703) by Pelikan it was conjectured that the box-counting dimension equals the uncertainty dimension for basin boundaries in typical dynamical systems. In this paper our main result is that the box-counting dimension, the uncertainty dimension and the Hausdorff dimension are all equal for the basin boundaries of one and two dimensional systems, which are uniformly hyperbolic on their basin boundary. When the box-counting dimension of the basin boundary is large, that is, near the dimension of the phase space, this result implies that even a large decrease in the uncertainty of the position of the initial condition yields only a relatively small decrease in the uncertainty of which basin that initial point is in.

T. Sauer, J.A. Yorke and M. Casdagli: EMBEDOLOGY. Preprint University of Maryland, January 1991. To appear in Journal of Statistical Physics.

ABSTRACT. Mathematical formulations of the embedding methods commonly used for the reconstruction of attractors from data series are discussed. Embedding theorems, based on previous work by H. Whitney and F. takens, are established for compact subsets A of the k-dimensional Euclidian space $\mathbb{R}^k$. If n is an integer larger than twice the box-counting dimension of A, then almost every map from $\mathbb{R}^k$ to $\mathbb{R}^n$, in the sense of prevalence, is one-to-one on A, and moreover is an embedding on smooth manifolds contained within A. If A is a chaotic attractor of a typical dynamical system, then the same is true for almost every delay-coordinate map from $\mathbb{R}^k$ to $\mathbb{R}^n$.

These results are extended in two other directions. Similar results are proved in the more general case of reconstructions which use moving averages of delay coordinates. Secondly, information is given on the self-intersection set that exists when n is less than or equal to twice the box-counting dimension of A.

Z. You, E.J. Kostelich, and J.A. Yorke: CALCULATING STABLE AND UNSTABLE MANIFOLDS. Preprint University of Maryland, January 1991. To appear in International Journal of Bifurcation and Chaos.

ABSTRACT. A numerical procedure is described for computing the successive images of a curve under a diffeomorphism of the n-dimensional Euclidian space. Given a tolerance ε, we show how to rigorously guarantee that each point on the computed curve lies no further than a distance ε from the "true" image curve. In particular, if ε is the distance between adjacent points (pixels) on a computer screen, then a plot of the computed curve coincides with the true curve within the resolution of the display. A second procedure is described to minimize the amount of computation of parts of the curve that lie outside a region of interest. We apply the method to compute the one-dimensional stable and unstable manifolds of the Hénon and Ikeda maps, as well as a Poincaré map for the forced damped pendulum.

UNIVERSITY OF MARYLAND AT COLLEGE PARK

INSTITUTE FOR PHYSICAL SCIENCE AND TECHNOLOGY

September 5, 1991

Dr. Arje Nachman
AFOSR/NM, Bldg. 410
Bolling Air Force Base
Washington, DC 20332-6448

Dear Dr. Nachman:

AFOSR-89-0401

Please find enclosed 6 copies of the final scientific report on the grant ~~AFOSR-01-5-2822~~5 for the period ending November 30, 1990 as well as the referenced manuscripts and preprints.

We will be happy to forward any further information on this grant which you may require

Sincerely,

James A. Yorke
Principal Investigator

Enclosures

cc: Ms. Evan Crierie
Office of Research Administration

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